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Characterization of sound radiation by unresolved scales of motion in computational aeroacoustics

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Abstract

Evaluation of the sound sources in a high Reynolds number turbulent flow requires time-accurate resolution of an extremely large number of scales of motion. Direct numerical simulations will therefore remain infeasible for the forseeable future; although current large eddy simulation methods can resolve the largest scales of motion accurately, they must leave some scales of motion unresolved. A priori studies show that acoustic power can be underestimated significantly if the contribution of these unresolved scales is simply neglected. In this paper, the problem of evaluating the sound radiation properties of the unresolved, subgrid-scale motions is approached in the spirit of the simplest subgrid stress models: the unresolved velocity field is treated as isotropic turbulence with statistical descriptors evaluated from the resolved field. The theory of isotropic turbulence is applied to derive formulas for the total power and the power spectral density of the sound radiated by a filtered velocity field. These quantities are compared with the corresponding quantities for the unfiltered field for various filter types and widths.

1. Introduction

The evaluation of the sound sources in high Reynolds number turbulent flows presents a fundamental problem for computational aeroacoustics. At one extreme, if a turbulence transport model is used to compute single-point single-time moments of the turbulence, the uncertainty of the turbulence model is compounded by the uncertainty of modeling the two-point two-time statistics required to evaluate the sound source. At the other extreme, the spatial resolution requirements of the direct numerical simulation of statistically steady turbulent flows are vastly increased by the requirements of aeroacoustics since the sound source depends on time correlations (Chapman [1]). Direct calculation of time correlations requires that the entire flow history be stored; this would impose prohibitive storage requirements even to compute the sound radiated by a model flow like isotropic turbulence. Further difficulties are posed if the acoustic field is to be resolved by direct numerical simulation as well (Colonius [2]; Piomelli et al. [3]).

A practical compromise appears to be emerging in which the sound sources are computed by large eddy simulation (LES) and are propagated to the far field by an acoustic analogy (Lighthill [4]; Ffowcs-Williams and Hawking [5]) or by solution of the linearized Euler equations (Béchara et al. [6]; Bastin et al. [7]; Bailly and Juvé [8]).

If the sound source is computed by LES, then the acoustic calculation will evaluate the sound radiated by the resolved velocity field alone. It can be anticipated that this will result at least in the suppression of high-frequency sound. A priori studies (Piomelli et al. [3]; Witkowska et al. [9]; Seror et al. [10]) suggest that this type of numerical sound suppression can be

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significant and motivate the present theoretical study of the relationship between the sound radiated by the exact velocity field and the sound radiated by the filtered velocity field.

This problem is addressed in the spirit of the simplest ideas used in subgrid stress modeling. Namely, we invoke Kolmogorov's theory of the universality of the small scales of motion in turbulence (Batchelor [11]) and assume that the unresolved scales can be modeled as isotropic turbulence with statistical descriptors computed from the resolved velocity field. In many problems, the assumption of isotropic turbulence may be inaccurate or inappropriate; nevertheless, this theory underlies many current LES models and is a reasonable starting point. Using the theory of the space-time properties of isotropic turbulence, we can construct models of the exact and filtered velocity fields and compare the sound radiated by both fields as a function of filter width. A different approach to subgrid-scale sound radiation is proposed by Seror et al. ([10]). The present work differs from this primarily in the emphasis on time correlation modeling.

The present analysis is closely related to another modeling method in computational aeroacoustics, the 'stochastic synthesis' of the subgrid motions (Béchara et al. [6]; Bailly et al. [8,12]). Like these methods, the present analysis depends on a model for the two-point two-time properties of the subgrid motions. Thus, our model (Eqs. (10) and (11) below), could be used as the statistical descriptor required to synthesize the subgrid sound sources.

Precise comparisons with existing a priori studies (Piomelli et al. [3]; Witkowska et al. [9]; Seror et al. [10]) are hampered by the very low Reynolds numbers of the direct simulations; the present analysis is appropriate when a Kolmogorov inertial range exists. Comparisons based on the non-universal spectra obtained in low Reynolds number simulations is possible, but less tractable analytically. This issue is discussed later.

2. The exact and filtered sound source

Denote the exact fluctuating velocity field by $\mathbf{u}(\mathbf{x},t)$, and the filtered field by $\bar{\mathbf{u}}(\mathbf{x},t)$. In homogeneous turbulence, the spatial Fourier transform of the resolved velocity field has the general form

$$\bar{\mathbf{u}}(\mathbf{k},t) = \mathbf{u}(\mathbf{k},t)Z(\kappa), \tag{1}$$

where Z, the Fourier transform of the filter function, is written as a function of the dimensionless wavenumber $\kappa = k/k_L$; k_L is related to the filter width Δ by

$$k_L = 2\pi/\Delta. \tag{2}$$

Only the sharp Fourier cutoff and the top-hat filters will be treated explicitly here. For the sharp Fourier cutoff filter,

$$Z(\kappa) = \begin{cases} 1 & \text{if } \kappa \leqslant 1, \\ 0 & \text{if } \kappa > 1 \end{cases} \tag{3}$$

and for the top-hat filter,

$$Z(\kappa) = \frac{\sin(\pi\kappa/4)}{\pi\kappa/4}. \tag{4}$$

The filters are normalized so that

$$\int_0^\infty \mathrm{d}\kappa\, Z(\kappa)^2 = 1 \tag{5}$$

for both Eqs. (3) and (4).

Lighthill's [4] formula for the acoustic pressure fluctuations in the far-field is

$$p(\mathbf{x},t) = \frac{1}{4\pi c^2}\frac{x_i x_j}{x^3}\int_V \mathrm{d}\mathbf{y}\, \ddot{T}_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{c}\right), \tag{6}$$

where the form of the Lighthill tensor for sound radiation by subsonic flow

$$T_{ij}(\mathbf{y},t) = \rho u_i(\mathbf{y},t)u_j(\mathbf{y},t) \tag{7}$$

will be used. In equations (6) and (7), V denotes the source region, ρ is the mean far-field density, c is the speed of sound in the far-field, and the vector $\mathbf{x}$ connects the measurement point to some representative point in the source region.

The sound radiated by the filtered velocity field has the same form as Eq. (6), but with the Lighthill tensor equation (7) computed from the resolved, rather than the exact velocity:

$$\overline{T}_{ij}(\mathbf{y},t) = \rho\bar{u}_i(\mathbf{y},t)\bar{u}_j(\mathbf{y},t). \tag{8}$$

Kraichnan [13] gave the far-field acoustic power spectral density as

$$p(\omega) = \pi\left(\omega^4/2c^8\right)\left\langle\left|n_i n_j T_{ij}(\omega\mathbf{n}/c,\omega)\right|^2\right\rangle, \tag{9}$$

where $\mathbf{n}$ is the unit vector in the direction of $\mathbf{x}$, ω is the frequency of the radiated sound, and $T_{ij}(\mathbf{k},\omega)$ is the space-time Fourier transform of the fluctuating quantity $T_{ij}(\mathbf{x},t)$. $T_{ij}(\mathbf{k},\omega)$ can be written in terms of the space-time Fourier transform of the velocity field as

$$T_{ij}(\mathbf{k},\omega) = \rho\int \delta(\mathbf{k}-\mathbf{p}-\mathbf{q})\,\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{q}\int_{\infty}^{\infty}\mathrm{d}\omega'\,u_i(\mathbf{p},\omega-\omega')u_j(\mathbf{q},\omega'). \tag{10}$$

It is necessary to express the result of Eq. (9) in terms of statistics of the velocity field. To this end, replace T_{ij} in Eq. (9) by result in Eq. (10). Close the resulting fourth-order velocity moment by quasinormality, shown to be reasonably accurate for this application by Zhou et al. [14], and use the correlation function for isotropic turbulence

$$\left\langle u_i(\mathbf{k},\omega)u_j(-\mathbf{k},-\omega)\right\rangle = \frac{1}{4\pi k^2}E(k)R(k,\omega)\left[\delta_{ij}-k_ik_jk^{-2}\right], \tag{11}$$

where $E(k)$ is the energy spectrum and $R(k,\omega)$ is defined as follows. For homogeneous, isotropic, time-stationary turbulence, define the Fourier transform

$$\frac{1}{(2\pi)^4}\int \mathrm{d}\,\mathbf{r}\,\mathrm{d}t\left\langle u_i(\mathbf{x},s)u_j(\mathbf{x}+\mathbf{r},s+t)\right\rangle \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}+i\omega t} = Q(k,\omega)\left[\delta_{ij}-\frac{k_ik_j}{k^2}\right]. \tag{12}$$

Then

$$Q(k,\omega) = \frac{1}{4\pi k^2}E(k)R(k,\omega), \tag{13}$$

where

$$E(k) = 4\pi k^2\int_{-\infty}^{\infty}\mathrm{d}\omega\,Q(k,\omega), \tag{14}$$

so that $R(k,\omega)$ is defined as

$$R(k,\omega) = 4\pi k^2 Q(k,\omega)/E(k). \tag{15}$$

It can be shown (Rubinstein and Zhou [15]) that Eq. (9) implies that sound is radiated only by interactions between incompressible modes nearly of the type $u_i(\mathbf{k},t)$ and $u_j(-\mathbf{k},t)$. This approximation treats the sound waves as infinitely long; equivalently, it ignores the so-called 'retarded time effect'. Introducing this approximation, the result (Rubinstein and Zhou [16]) is

$$p(\omega) = C\frac{\omega^4}{Vc^5}\int_0^{\infty}\mathrm{d}k\,E(k)^2k^{-3}\hat{R}(k,\omega), \tag{16}$$

where $\hat{R}$ denotes the frequency convolution

$$\hat{R}(k,\omega) = \int_{-\infty}^{\infty}\mathrm{d}\omega'\,R(k,\omega-\omega')R(k,\omega'). \tag{17}$$

3. Sound radiation by the filtered velocity field

To complete the calculation of the far-field acoustic power spectral density function using equation (16), models for the energy spectrum $E(k)$ and the time correlation function $R(k,\omega)$ are needed (Bailly and Juvé [8]; Zhou et al. [17]; Woodruff et al. [18]).

Our previous work (Zhou and Rubinstein [19]) discusses the issue of time correlations in detail. It is concluded that sound radiation is determined by Eulerian time correlations, for which turbulence theory (Kraichnan [20]; Tennekes [21]; Kaneda [22]) demonstrates the similarity form

$$R(k,\omega) = r\left(\frac{\omega}{Vk}\right) \tag{18}$$

for inertial range scales, where V is the rms of the fluctuating velocity. This similarity form has also been used recently by Bailly and Juvé [8], where V is referred to as the 'Heisenberg velocity'.

Substituting the similarity form equation (18) in Eq. (17) defines the function

$$\hat{r}\left(\frac{\omega}{Vk}\right) = \int_{-\infty}^{\infty} \mathrm{d}\omega' \, r\left(\frac{\omega - \omega'}{Vk}\right) r\left(\frac{\omega'}{Vk}\right) \tag{19}$$

in terms of which we have

$$p(\omega) = C\frac{\omega^4}{Vc^5}\int_0^{\infty} \mathrm{d}k \, E(k)^2 k^{-3} \hat{r}\left(\frac{\omega}{Vk}\right). \tag{20}$$

Next, let $E(k)$ denote the Kolmogorov spectrum

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}, \quad k_0 \leqslant k \leqslant k_d, \tag{21}$$

where k_0 is the inverse integral scale and k_d is proportional to the inverse Kolmogorov scale. The subsequent calculation will show that the exact form of the spectrum in the region of large scales $k \leqslant k_0$ is not important for the calculation of subgrid sound. Similarly, the spectrum in the dissipation range with $k \geqslant k_d$ can be neglected because of its insignificant energy content. Substituting Eq. (21) in Eq. (20),

$$p(\omega) = C\frac{\varepsilon^{4/3}}{c^5}\omega^4 \int_0^{\infty} \mathrm{d}k \, k^{-19/3} \hat{r}\left(\frac{\omega}{Vk}\right). \tag{22}$$

If the time correlation function $\hat{r}$ decays sufficiently rapidly at ∞, this integral is finite for k near 0; consequently, as noted earlier, the precise form of $E(k)$ for small k is not needed for this calculation. Regardless of the functional form of $\hat{r}$, changing variables in Eq. (22) to $k^* = Vk/\omega$ shows that for large ω,

$$p(\omega) \approx C\frac{V^{13/3}\varepsilon^{4/3}}{c^5}\omega^{-4/3}. \tag{23}$$

Next, consider the sound radiated by the filtered velocity field. Denote its power spectral density by $p_L(\omega)$ to indicate the dependence on the filter scale k_L. We noted in the previous section that ignoring the retarded time effect means that only mode pairs of the form $u_i(\pm\mathbf{k}, t)$ interact to radiate sound (Rubinstein and Zhou [15]). If so, the sound radiated by the filtered velocity field $\bar{u}_i(\mathbf{k}, t)$ is found from Eq. (22) by multiplying by the square of the appropriate filter function

$$p_L(\omega) = C\frac{\omega^4}{Vc^5}\int_0^{\infty} \mathrm{d}k \, E(k)^2 k^{-3} \hat{r}\left(\frac{\omega}{Vk}\right) Z\left(\frac{k}{k_L}\right)^2 \tag{24}$$

whence for the Kolmogorov spectrum

$$p_L(\omega) = C\frac{\varepsilon^{4/3}}{c^5}\omega^4 \int_0^{\infty} \mathrm{d}k \, k^{-19/3} \hat{r}\left(\frac{\omega}{Vk}\right) Z\left(\frac{k}{k_L}\right)^2. \tag{25}$$

Analytical results will require assuming a specific functional form for the time correlation function. We note three forms:

1. Kraichnan's [23] result
$$r(x) = \frac{J_1(x)}{x}; \tag{26}$$
2. the Markovian approximation
$$r(x) = \exp\bigl(-|x|\bigr); \tag{27}$$
3. the Gaussian approximation
$$r(x) = \exp\bigl(-x^2\bigr), \tag{28}$$

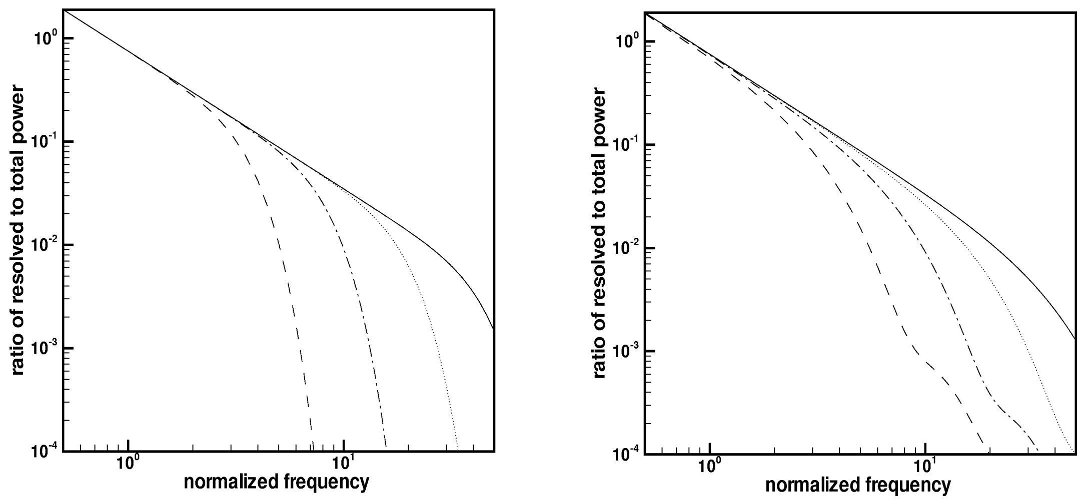

Fig. 1. Effect of filtering on resolved sound radiation. The power spectral density of sound radiated by filtered velocity field is shown, on the left for the sharp Fourier cutoff filter, on the right for the top hat filter. Resolutions for the Fourier cutoff filter are $\omega_0/\omega_L = k_0/k_L = 0.9^4 \approx 0.43$ (dashed), $0.9^8 \approx 0.18$ (dot-dashed), $0.9^{16} \approx 0.08$ (dotted), $0.9^{32} \approx 0.03$ (solid). The corresponding resolutions for the top-hat filter are described under equation (2).

where x is the similarity variable $x = \alpha\omega/Vk$. The constant α should be chosen to match the second-order Taylor coefficient of Eqs. (26)–(28) to the short-time expansion of the Navier–Stokes equations following the analysis of Kaneda [22].

The analytically most convenient form is the Gaussian of Eq. (28), which is tentatively adopted here in order to illustrate the derivation of a theory of subgrid-scale sound. Regardless of which of Eqs. (26)–(28) is used, the final result can be written in the form

$$p_L(\omega) = C\frac{\varepsilon^{4/3}V^{13/3}}{c^5}\omega^{-4/3}F\left(\frac{\omega}{Vk_L}\right). \tag{29}$$

For Eq. (28), the function F can be expressed as an incomplete gamma function.

The effect of filter width on the spectrum of sound radiated by the filtered velocity field is shown in Fig. 1 in which the dimensionless power spectrum is plotted as a function of normalized frequency ω/ω_0 where $\omega_0 = Vk_0$ is the frequency integral scale. The filter wavelength k_L is converted to a frequency ω_L through the relation

$$\omega_L = Vk_L \tag{30}$$

and the spectrum in Eq. (29) is plotted for the values $\omega_0/\omega_L = k_0/k_L = 0.9^4 \approx 0.43, 0.9^8 \approx 0.18, 0.9^{16} \approx 0.08, 0.9^{32} \approx 0.03$ ranging from extremely coarse to very fine resolution of the fluctuating field. The slope changes in the curves corresponding to the top-hat filter are caused by the side-lobes of the corresponding filter function equation (4). The most conspicuous effect of coarsening the resolution is suppression of high frequency sound, although there is some effect at all frequencies.

Fig. 1 clearly suggests that the top-hat filter better resolves the high-frequency sound, although the spectral attenuation exists for both filters at sufficiently high frequency. The difference between the filters is shown more clearly in Fig. 2 which compares two corresponding curves from Fig. 1. The top-hat filter slightly reduces the power in the largest scales, but gives a better representation of the sound radiated by the smaller scales. The reason is that the Fourier transform Z for the cut-off filter, defined in Eq. (2), falls off with wavenumber gradually rather than abruptly. The algebraic decay of the peaks in Eq. (4) mitigates the attenuation of the small scales.

Perhaps more immediately important is the effect of filter width on the computed total acoustic power. The exact total acoustic power is

$$P = \int_0^\infty d\omega\, p(\omega) \tag{31}$$

and the total power radiated by the resolved scales is

$$P_L = \int_0^\infty d\omega\, p_L(\omega). \tag{32}$$

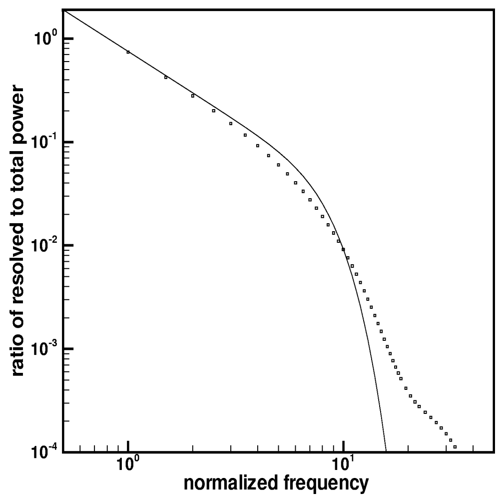

Fig. 2. Effect of filter type on the power spectrum of sound radiated by the resolved velocity field. The spectra from Fig. 1 at resolution $\omega_0/\omega_L = 0.9^8$ are compared for the cut-off filter (solid line) and the top-hat filter (symbols). The top-hat filter transfer function falls off gradually at large frequency, consequently the radiation by small scales is better represented.

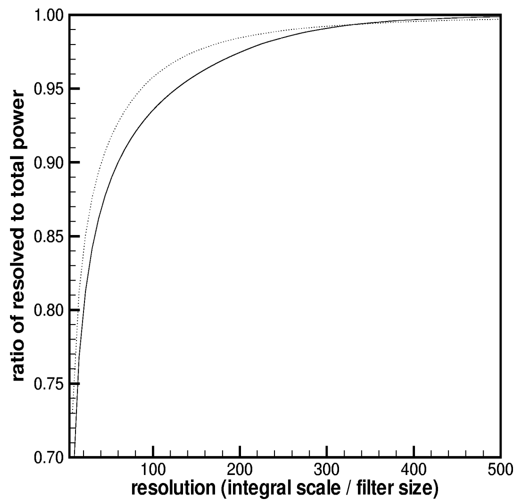

Fig. 3. Effect of filter type on the total acoustic power radiated by the resolved velocity field. The ratio of resolved acoustic power to total acoustic power is compared for the cut-off filter (solid line) and the top-hat filter (dotted line). Because of the relatively slow fall-off of the power spectral density with frequency, the top-hat filter is superior in capturing the total acoustic power at moderate resolution.

Using Eq. (29), the ratio of resolved to total acoustic power is

$$\frac{P_L}{P} = \frac{1}{3F(0)} \int_1^\infty \mathrm{d}\tilde{\omega} (\tilde{\omega})^{-4/3} F\left(\tilde{\omega}\frac{\omega_0}{\omega_L}\right). \tag{33}$$

This function is shown for both filter types in Fig. 3 as a function of the variable $\omega_L/\omega_0 = k_L/k_0$, the ratio of the integral scale to the filter size. Although the resolution rises extremely rapidly with resolution for low resolutions, rather large resolutions are needed to resolve the total power to within a few percent. From a practical standpoint, Fig. 3 suggests that a filter size of about 0.05 times the integral scale gives the total power to within about 1 dB.

All of these estimates of the acoustic power in the subgrid scales must be considered preliminary because of the assumption of the Gaussian time correlation function which has been chosen largely for analytical convenience.

The problem of acoustic power radiated by subgrid scales has been addressed in a priori studies by Piomelli et al. [3] for the model problem of channel flow and by Witkowska et al. [9] for the problem considered here of isotropic turbulence. The latter study considers both forced, steady turbulence and the more commonly studied problem of decaying turbulence. Comparison with these results requires some care because of the limited Reynolds numbers of the simulations. This comparison must be based on results like Eq. (24) expressed in terms of a numerically given spectrum $E(k)$ instead of the Kolmogorov spectrum used here.

4. Conclusions

The present work is a first step to an analytical theory of the subgrid contribution to radiated sound. It shows how the theory of isotropic turbulence can be applied to derive a theory of subgrid-scale sound radiation. Refinement of this model will require closer investigation of the time correlation function, which is the key ingredient of our analysis.

The close connection of this work to the method of stochastic synthesis advanced by Bailly et al. [8,12] and by Béchara et al. [6] was noted earlier. Although this application has not been developed explicitly here, the present theory is based on a two-point two-time model of the subgrid scales which could also be used to synthesize the subgrid-scale motions.

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